PROBABILITIES FOR 6/10/2012 CHALLENGE

The protocol for this challenge was as follows: In each trial, the subject was presented with four photographs. He was then asked to match those photographs to identical photographs in sealed envelopes. This was repeated for three trials.

Okay, first of all, we need the probabilities for getting particular numbers of matches in *one* trial. There are 4! = 24 possible orders in which the four pictures can be arranged; only one of those orders will match all four of the pictures in the envelope, so the probability of getting all four correct is 1/24. It is impossible to match exactly three correctly in any trial; if you get three correct, you necessarily must have also gotten the fourth. There is only one way to get only the first two correct: if the pictures are labeled A, B, C, and D and the target order is ABCD, then the only order to match only the first two is ABDC. But we don't care *which* two we get correct, so we have to multiply this by the number of pairs we can choose out of four, which mathematically is ${}_4C_2 = \frac{4!}{2!\cdot 2!} = 6$. (Of course, the twenty-four possible combinations are a small enough number that we could also arrive at this result by simply counting the combinations with two matches.) The probability of getting exactly two correct is therefore 6/24=1/4. By similar means, we find that the probability of getting exactly one correct is 8/24=1/3, and finally the probability of getting none correct is 1 – (1/3+1/4+1/24) = 3/8.

Now, to find out the probability of getting X matches correct out of 12 according to the given protocol (three trials of four pictures each), there are two steps.

First, we have to count all the possible ways these matches can be distributed among the three trials. To get exactly six matches correct, for instance, he could have gotten two correct in each trial; or he could have gotten none in one trial, two in another, and four in the third; or he could have gotten one right in two trials and four in the third. That is, the correct matches could have been divided among the trials as 2/2/2, 1/1/4, or 0/2/4. (Note that 0/3/3 and 1/2/3 are not possibilities because, as noted above, it is impossible to get exactly three correct in any trial.)

Second, for each combination, we have to count the probability of that combination. To do this, we also have to take into account the fact that we don't care about the order. For 2/2/2, for instance, the probability of getting exactly two correct in the first trial is 1/4, the probability of getting exactly two correct in the second trial is 1/4, and the probability of getting exactly two right in the third trial is 1/4, for a total probability of (1/4)(1/4)(1/4) = 1/64. For 1/1/4, multiplying the probabilities of getting one right in the first trial, one right in the second, and four in the third gives (1/3)(1/3)(1/24)... but because there are three possible orders (1/1/4, 1/4/1, or 4/1/1), we multiply that by 3 to get 3·(1/3)(1/3)(1/24) = 1/72. For 0/2/4, we multiply the respective probabilities to get (3/8)(1/4)(1/24), but we have to multiply that by *6* because of the six possible orders (0/2/4, 0/4/2, 2/0/4, 2/4/0, 4/0/2, or 4/2/0) to get 6·(3/8)(1/4)(1/24) = 3/128. In general, when the numbers correct in all three trials are different (as in 0/2/4), we multiply by 6; when the numbers correct in two trials are correct and the third is different (as in 1/1/4), we multiply by 3; when the numbers correct are the same in all three trials (as in 2/2/2), we don't have to multiply by anything (or, equivalently, we multiply by 1).

So, let's apply this method to each combination. See the table on the next page for details.

Number correct	Combinations	Probability calculation	Result
0	0/0/0	(3/8)(3/8)(3/8)	0.052734
1	0/0/1	3·(3/8)(3/8)(1/3)	0.140625
2	0/1/1 or 0/0/2	3·(3/8)(1/3)(1/3) + 3·(3/8)(3/8)(1/4)	0.230469
3	1/1/1 or 0/1/2	(1/3)(1/3)(1/3) + 6·(3/8)(1/4)(1/3)	0.224537
4	1/1/2, 0/2/2, or 0/0/4	3·(1/3)(1/3)(1/4) + 3·(3/8)(1/4)(1/4) + 3·(3/8)(3/8)(1/24)	0.171224
5	1/2/2 or 0/1/4	3·(1/3)(1/4)(1/4) + 6·(3/8)(1/3)(1/24)	0.093750
6	2/2/2, 1/1/4, or 0/2/4	(1/4)(1/4)(1/4) + 3·(1/3)(1/3)(1/24) + 6·(3/8)(1/4)(1/24)	0.052951
7	1/2/4	6·(1/3)(1/4)(1/24)	0.020833
8	2/2/4 or 0/4/4	3·(1/4)(1/4)(1/24) + 3·(3/8)(1/24)(1/24)	0.009766
9	1/4/4	3·(1/3)(1/24)(1/24)	0.001736
10	2/4/4	3·(1/4)(1/24)(1/24)	0.001302
11	None possible	0	0.000000
12	4/4/4	(1/24)(1/24)(1/24)	0.000072